1. Find the $x$ and $y$ intercepts for the function $f(x) = x^3 - 9x$.

Solution: To find $x$ intercepts, solve $f(x) = 0$:

$x^3 - 9x = 0 \Rightarrow x = 0, \pm 3$. To find the $y$ intercept, substitute $x = 0$: $f(0) = 0$. Hence the $y$ intercept is $y = 0$.

2. Find the domain of:

(a) $f(x) = \sqrt{-x^2 - 4x + 5}$

Solution: We must have $-x^2 - 4x + 5 \geq 0$ in order for $\sqrt{-x^2 - 4x + 5}$ to be defined. Thus $(-x + 1)(x + 5) \geq 0$, giving us $-5 \leq x \leq 1$.

(b) $g(t) = \ln(4t - 3)$

Solution: We must have $4t - 3 > 0$ in order for $\ln(4t - 3)$ to be defined. This yields $t > \frac{3}{4}$.

(c) $h(x) = \frac{1}{x^3 + 3x^2 - x - 3}$

Solution: We must have $x^3 + 3x^2 - x - 3 \neq 0$ in order for $\frac{1}{x^3 + 3x^2 - x - 3}$ to be defined. $x^3 + 3x^2 - x - 3 \neq 0 \Rightarrow (x^2 - 1)(x + 3) \neq 0$, yielding $x \neq \pm 1$ and $x \neq -3$. Thus the domain is all real numbers except $x = \pm 1$ and $x = -3$.

3. Simplify the expression. Write your answer using positive rational exponents. $\left( \frac{2}{\sqrt{x^5}} \right) \left( \sqrt[3]{4x} \right)$

Solution: $\left( \frac{2}{\sqrt{x^5}} \right) \left( \sqrt[3]{4x} \right) = (2x^{-5/2})(2^{2/3}x^{1/3}) = \frac{2^{5/3}}{x^{13/6}}$

4. If we begin with the graph of $f(x) = \sqrt{x}$, shift 4 units to the right, shrink vertically by a factor of $\frac{1}{2}$, and shift upward 10 units, write an equation for the transformed graph.

Solution: The transformed graph is $g(x) = \frac{1}{2}\sqrt{x - 4} + 10$. 

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5. Solve for $x$: $\log(x + 2) + \log(x - 1) = 1$.
Solution: Use properties of logarithms:

$$\log(x + 2) + \log(x - 1) = 1 \Rightarrow \log((x + 2)(x - 1)) = 1 \Rightarrow (x + 2)(x - 1) = 10.$$ This yields $x = -4$ and $x = 3$. Since $x = -4$ is not in the domain, $x = 3$ is the only solution.

6. Factor completely: $3x^2(4x^2 + 1)^8 + 64x^4(4x^2 + 1)^7$.
Solution: $3x^2(4x^2 + 1)^8 + 64x^4(4x^2 + 1)^7 = x^2(4x^2 + 1)^7(76x^2 + 3)$.

7. How far from the base of an 18 foot tall pole must a person be standing if the angle of elevation from the ground to the pole is $41^\circ$?
Solution: Refer to the figure below. We know that $\tan(41^\circ) = \frac{18}{x}$, thus $x = \frac{18}{\tan(41^\circ)}$, or $x = 18\cot(41^\circ)$.

8. Find $f \circ g$ if $f(x) = \frac{x}{x + 1}$ and $g(x) = \frac{2}{x}$. Simplify.
Solution: $f \circ g = \frac{2}{x + 1} = \frac{2}{2 + x}$.

9. Perform the indicated operation and simplify: $\frac{8}{x + 1} - \left( \frac{y}{z + 2} \div \frac{y - 4}{w} \right)$
Solution: After getting a common denominator of $(x + 1)(z + 2)(y - 4)$, we find that

$$\frac{8}{x + 1} - \left( \frac{y}{z + 2} \div \frac{y - 4}{w} \right) = \frac{8zy - 32z + 16y - 64 - ywx - yw}{(x + 1)(z + 2)(y - 4)}.$$ 

10. Solve for $x$: $e^{2x} - 2e^x - 3 = 0$.
Solution: $e^{2x} - 2e^x - 3 = (e^x)^2 - 2e^x - 3 = (e^x - 3)(e^x + 1)$. Solving $(e^x - 3)(e^x + 1) = 0$ yields $x = \ln 3$ as the only solution.
11. Find the equation of the line passing through the point $(5, 1)$ with slope 7. Next, find $y$ when $x = -4$.

Solution: The equation of the line is $y - 1 = 7(x - 5)$. Thus if $x = -4$, $y = -62$.

12. If $f(x) = \sqrt{x + 4}$, find and simplify $\frac{f(2 + h) - f(2)}{h}$.

$$
\frac{f(2 + h) - f(2)}{h} = \frac{\sqrt{6 + h} - \sqrt{6}}{h}.
$$

Multiply the numerator and denominator by $\sqrt{6 + h} + \sqrt{6}$ yields a simplified answer of $\frac{1}{\sqrt{6 + h} + \sqrt{6}}$.

13. Simplify $\frac{(x^2 y^4)^5 (x^3 y)^{-3}}{xy}$.

Solution: $\frac{(x^2 y^4)^5 (x^3 y)^{-3}}{xy} = \frac{x^{10} y^{20} x^{-9} y^{-3}}{xy} = y^{16}$.

14. Simplify $\sqrt[3]{a^3 b \sqrt[4]{64 a^4 b^2}}$.

Solution: $\sqrt[3]{a^3 b \sqrt[4]{64 a^4 b^2}} = \sqrt[3]{64 a^7 b^3} = 4a^2 b \sqrt[3]{a}$.

15. Perform the operations and simplify.

$$
\frac{x^2}{x^2 - x - 2} - \frac{4}{x^2 + x - 6} + \frac{x}{x^2 + 4x + 3}
$$

Solution: Factor the denominators:

$$
\frac{x^2}{(x - 2)(x + 1)} - \frac{4}{(x + 3)(x - 2)} + \frac{x}{(x + 3)(x + 1)}.
$$

After getting a common denominator of $(x+1)(x-2)(x+3)$, we obtain $\frac{x^3 + 4x^2 - 6x - 4}{(x + 1)(x - 2)(x + 3)}$.

16. Find all zero’s and vertical asymptotes for $f(x) = \frac{3x^2 - 14x - 5}{4x^2 - 17x - 15}$

Solution: Factor and simplify: $f(x) = \frac{3x + 1}{4x + 3}$. This yields a zero of $x = -\frac{1}{3}$ and a vertical asymptote of $x = -\frac{3}{4}$. 
17. If $\theta$ is in quadrant II and $\sin \theta = \frac{1}{7}$, what is $\cos \theta$?

Solution: Using the identity $\cos^2 \theta + \sin^2 \theta = 1$, we find that $\cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{1}{49}$. This gives us $\cos \theta = \pm \frac{\sqrt{48}}{7}$. Since $\theta$ is in quadrant II, $\cos \theta < 0$, hence $\cos \theta = -\frac{4\sqrt{3}}{7}$.

18. Use properties of logarithms to expand the expression $\ln \left( \frac{\sqrt{xy^5}}{(z + 1)^4} \right)$.

Solution: We know $\ln ab = \ln a + \ln b$; $\ln \frac{a}{b} = \ln a - \ln b$ and $\ln a^b = b \ln a$. Using these properties, we obtain

$$\ln \left( \frac{\sqrt{xy^5}}{(z + 1)^4} \right) = \frac{1}{2} \ln x + 5 \ln y - 4 \ln(z + 1).$$

19. Evaluate $\sec \frac{2\pi}{3} - \tan \frac{\pi}{6}$.

Solution: $\sec \frac{2\pi}{3} - \tan \frac{\pi}{6} = -2 - \frac{1}{\sqrt{3}}$.

20. If we begin with a rectangle with length 5 inches and width 4 inches, then increase the length by 8%, what is the change in area?

Solution: We begin with a rectangle of area 20 square inches. If we increase the length by 8%, then our new length is 5.4 inches. This gives a new area of 21.6 square inches. This gives a total change in area of 1.6 square inches.

21. Evaluate $f(2) - f(-3)$ If

$$f(x) = \begin{cases} 
  x^3 + 1 & , \text{ if } x > 1 \\
  2x^2 - 3 & , \text{ if } x \leq 1 
\end{cases}$$

Solution: $f(2) = 9$ and $f(-3) = 15$. Thus $f(2) - f(-3) = -6$.

22. Simplify the expression $\frac{\cos^2 \theta}{1 + \sin \theta}$.

Solution: Use the identity $\cos^2 \theta = 1 - \sin^2 \theta$. Then after using the difference of two squares, we obtain a simplified answer of $1 - \sin \theta$. 
23. Evaluate \( \log_4 \frac{1}{\sqrt[3]{16}} \).

Solution: Using the fact that \( \log_b b^x = x \), we find that
\[
\log_4 \frac{1}{\sqrt[3]{16}} = -\frac{2}{3}.
\]

24. Simplify \( \frac{\frac{a}{b} - b}{\frac{1}{b^2} + a} \).

Solution: Multiply the numerator and denominator by \( ab^3 \). This gives us
\[
\frac{\frac{a}{b} - b}{\frac{1}{b^2} + a} = \frac{b^3 - ab^4}{a + a^2b^3}.
\]

25. A bacteria culture contains 1200 bacteria and doubles every day. How many hours will it take the culture to reach 10000 bacteria?

Solution: Use the exponential growth model: \( y(t) = y_0 e^{kt} \), where \( y(t) \) is the size of the population at time \( t \) and \( y_0 \) is the initial size of the population. We know that \( y_0 = 1200 \), hence \( y(t) = 1200 e^{kt} \). Using the fact that the population doubles every day, we find that \( k = \ln 2 \). Now we will solve \( y(t) = 10000 \) for \( t \). This gives
\[
t = \frac{\ln(25/3)}{\ln 2} \text{ days}, \quad \text{hence } t = 24 \left( \frac{\ln(25/3)}{\ln 2} \right) \text{ hours}.
\]